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Study of the surface conductance coefficient in a metallic bar

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Abstract

The surface conductance coefficient H of a cylindrical iron bar is calculated from experimental measurements of the temperature distribution in the steady state and during cooling. In the steady state, H can be considered constant, with fluctuations within statistical error. In cooling, there are two H coefficients: one governing the sharper temperature fall over short times, and the other governing the smoother fall over longer times.

INTRODUCTION

In the interior of a solid, the temperature T is assumed to be a continuous function of position (x, y, z) and time t, and this is assumed also to hold for the first differential coefficient with respect to time and for the first and second differential coefficients with respect to position. These assumptions are not made at the boundary of the solid or at the instant when heat flow is taken to start, and it is necessary to determine the expressions for the initial conditions and boundary conditions. The boundary condition of linear heat transfer at the surface [1] into a medium at T_0 corresponds to Newton's law of cooling

$$\text{surface flux} = H(T - T_0) \tag{1}$$

Here, H is a constant called the surface conductance or the coefficient of surface heat transfer, its inverse being the surface thermal resistance per unit area. If H tends to zero, there can be no flux across the surface, whereas if H tends to infinity, the boundary conditions can be a prescribed surface temperature which may be constant, or a function of time, or position, or both. Although this latter is the easiest boundary condition to work with and that which has been most studied, in practice it is often difficult to prescribe surface temperature, and a linear heat transfer boundary condition, such as eqn. (1), may better represent real situations [1].

Equation (1) approximately describes several different heat flow problems, such as (a) forced convection, in which a fluid at T_0 is forced rapidly past the surface of the solid, (b) combined forced convection and evapora-

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tion, and (c) the existence of a thin skin of a poor conductor, such as scale, grease or oxide, on the surface of a body. In most practical cases, however, the heat flow from the surface is non-linear and eqn. (1) is not valid. Examples are (a) black-body radiation, which obeys the Stefan–Boltzmann fourth-power temperature law, and (b) in natural convection, where it is found experimentally that the rate of heat loss from the body is proportional, not to the temperature difference between the body and the surrounding fluid, but very nearly to the power 5/4. Nevertheless, for small ranges of temperature, this non-linear behaviour may be approximated by a linear law like eqn. (1).

In any case, the problem is that H is not necessarily constant because it depends on several factors, such as the nature of the material, the shape of the surface, the temperature of the body, and so on. Numerical values of H are to be found in various references, for example refs. 1 and 2, which for a few special cases allow a rough approximation of practical heat transfer to be made.

This paper describes an approach to determining the H coefficient from new experimental data on the temperature distribution in a metallic bar. Section 2 describes the mathematics by which H can be calculated from the steady state and cooling data. The experimental results and their analysis are given in Section 3, and a number of conclusions are presented in Section 4.

MATHEMATICAL TREATMENT

The broad mathematical treatment of heat transfer in a solid is applied in this section to the case under consideration, which is the linear heat flow for an isotropic homogeneous bar, where all thermal, physical and geometrical properties (such as the conductivity K, density ρ, specific heat c, perimeter p and cross section w) are constant in the range of temperature used.

The differential equation for the temperature distribution in a semi-infinite bar, with no internal source of heat, lying along the x-axis is given by [2]

$$\frac{\partial T}{\partial t} = \frac{K}{\rho c}\frac{\partial^2 T}{\partial x^2} - \frac{Hp}{c\rho w}(T - T_0) \tag{2}$$

where $K/c\rho$ has dimensions of length squared per unit time and is known as the thermal diffusivity α of the conducting material.

Steady state

If the flow of heat is invariant with respect to time, i.e. for the steady state, eqn. (2) reduces to

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} - \frac{Hp}{Kw}T = 0 \tag{3}$$

where the constant temperature of the surrounding medium is taken as the zero of the temperature scale. The solution of eqn. (3) is a single exponential given by

$$T = T_1 \, e^{-mx} \tag{4}$$

where T_1 is the constant temperature at the end of the bar and

$$m^2 = Hp/Kw \tag{5}$$

Non-steady state

Now let us consider the transient state, i.e. the heating or cooling of the body as a function of time. The simplest case is that in which the body has a large thermal conductivity and a correspondingly low heat-transfer coefficient. The heat flow to or from the body is controlled principally by the surface thermal resistance (convection resistance of ref. 2); the body is spacewise isothermal and the temperature varies only with time. In this case, eqn. (2) becomes

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{-Hp}{c\rho w} T \tag{6}$$

the solution of which is another exponential given by

$$T = T_i' \, e^{-m_c t} \tag{7}$$

where T_i' is the temperature of some point i of the bar at time zero and

$$m_c = Hp/c\rho w \tag{8}$$

A plot of eqn. (7) for $\ln(T/T_0)$ vs. t would yield a family of straight lines with m_c as a parameter. As can be seen, m_c does not depend on K, but may be modified usefully taking into account the diffusivity α and the ratio w/p, which, for most regular geometric shapes, is equal to the ratio L of the volume of the body to the surface area and is recognized as a significant dimension of the system. Thus, m_c can be rewritten as

$$m_c t = (HL/K)(\alpha t/L^2) = Bi \times Fo \tag{9}$$

where (HL/K) and $(\alpha t/L^2)$ are known as the Biot (Bi), and Fourier (Fo) moduli respectively. These moduli are dimensionless, and have the great advantage that, when used in the manner of eqn. (9), the temperature–time histories for all bodies of finite thermal conductivity can be reduced to a single universal plot for all values of the convection boundary conditions. Now, eqn. (7) may be plotted for $\ln(T/T_0)$ vs. Fo with Bi as a parameter.

For a cylinder of radius r_0, the value of L is $r_0/2$, thus

$$Bi = Hr_0/2K \tag{10}$$

and

$$Fo = 4at/r_0^2 \tag{11}$$

If H satisfies eqn. (1), i.e. does not depend on the temperature distribution regime (whether steady or non-steady state), eqns. (5) and (8) lead to the following relationship between m and m_c

$$m^2 = \rho c m_c/K \tag{12}$$

RESULTS AND DISCUSSION

As was seen in the previous section, if the temperature distribution in the bar obeys an exponential law for the steady state and cooling, the exponents m and m_c are dependent on H. The accuracy of the derived value of H will depend on the goodness of fit of the experimental data to the steady state and cooling laws.

The experimental results were obtained on a cylindrical iron bar of 155 cm length, 2.25 cm radius, 7.8 g cm^{-3} density, 0.452 J g^{-1} K^{-1} specific heat, and 0.803 J s^{-1} cm^{-1} K^{-1} thermal conductivity. The bar was insulated with asbestos cord and heated by a resistive coil at one end. The temperature distribution was measured by thermocouples inserted along the axis and connected to a data logger. The coordinate origin was taken to be 11 cm from the heated end, where the first thermocouple was placed. Taking into account the temperature gradient in the bar, the others were placed at 4, 10, 16, 25.5, 36, 51, 66, 81, 96, 116, and 136 cm from the first. The experimental procedure has been described in detail elsewhere [3].

Steady state

Table 1 shows the results for the nine selected steady states, labelled I–IX, covering a range of temperature of about 50 °C, where T_{1e} means the

TABLE 1

Values for the steady state fitting of the experimental results to an exponential law, eqn. (4)

Case	T_{1e}	T_{1t}	$-m$	$\Sigma(T_{ie} - T_{it})^2$	H
I	11.3	11.1	3.79	0.13	15.86
II	17.6	17.2	3.21	0.64	11.38
III	25.4	25.7	4.00	0.92	17.66
IV	26.0	26.1	4.13	0.28	18.83
V	28.6	28.7	3.55	0.23	13.91
VI	33.3	32.2	3.99	4.53	17.58
VII	34.9	34.9	3.71	0.35	15.20
VIII	47.4	44.4	3.40	27.17	12.76
IX	61.5	61.2	3.50	0.40	13.53

excess of the experimental temperature in the first thermocouple over room temperature. The full steady state temperature distribution for all the cases is available on request. The entry T_{1t} is the theoretical temperature at the first point when the experimental data are fitted to an exponential equation as eqn. (4), and m is the exponent. The values of H in the last column were calculated from eqn. (5).

The experimental results are perfectly fitted by a single exponential, as can be seen from the squares of the differences, with one small discrepancy in VIII caused by the first two values of the temperature. Nevertheless, additional fits to a double exponential and to power laws were performed for all the cases. Except for case VIII, in which one exponent is much greater than the other, the double exponential always reduces to a single exponential because the two exponents are the same and the sum of the coefficients gives the value T_{1t} with a standard deviation of ± 0.6, as in the single exponential fit. In all cases the power law was far from being a good fit.

Every value of m has a standard deviation of ± 0.06. The results are quite consistent with m being a constant with mean value $\bar{m} = 3.7 \pm 0.3$. As a consequence, the calculated values of H are also consistent with being constant, but the uncertainty is greater as a result of error propagation [4]. Every H has a standard deviation of ± 0.22, and the mean value of H from substituting $\bar{m}$ in eqn. (5) is $\bar{H} = 15.2 \pm 0.7$.

Non-steady state

Cooling was measured starting from the nine steady states described above. Table 2 lists a selection of the results: for two intermediate temperatures (cases III and VIII) and the highest and the second lowest temperatures (cases IX and II). The entries T_1' and m_c are the parameters of eqn. (7) to fit a single exponential to the measurements of cooling at the first point along the bar. The entry N is the number of measurements made during the time of cooling t for each point.

The values of m_c obtained from the fit show that they can be considered as independent of temperature, giving the same value within statistical error. The mean value is $\bar{m}_c = 0.24 \pm 0.01$. From this value of $\bar{m}_c$, $\bar{H}$ can now be

TABLE 2

Values for the cooling of the first point of the bar for the cases chosen. The experimental results have been fitted to an exponential law, eqn. (7)

Case	$T_1' \pm \sigma$	$-(m_c \pm \sigma)\times 10^{-3}$	$\Sigma(T_{ie} - T_{it})^2$	N	$t \times 10^3$ (s)
II	18.0±0.7	0.238±0.015	7.52	43	9.72
III	24.4±0.8	0.223±0.011	16.91	31	12.00
VIII	46.5±0.6	0.241±0.006	67.51	48	20.82
IX	58.1±0.9	0.241±0.006	68.79	26	19.20

TABLE 3

The same as Table 2, but for a double exponential law, eqn. (13)

Case	$T_1' \pm \sigma$	$-(m' \pm \sigma)\times 10^{-3}$	$T_1'' \pm \sigma$	$-(m'' \pm \sigma)\times 10^{-3}$	$\Sigma(T_{ie} - T_{it})^2$
II	11.8 ± 0.5	0.42 ± 0.03	7.35 ± 0.03	0.121 ± 0.007	1.15
III	19.3 ± 0.8	0.37 ± 0.03	6.66 ± 0.01	0.078 ± 0.004	0.87
VIII	23.4 ± 0.4	0.57 ± 0.02	27.02 ± 0.02	0.156 ± 0.003	1.64
IX	37.6 ± 1.2	0.45 ± 0.03	24.33 ± 0.01	0.134 ± 0.002	0.19

calculated using eqn. (8). The result is $\bar{H} = 11.6 \pm 1.0$. The discrepancy between this value of H and that obtained in the steady state (15.2) is considerable, and eqn. (12) is consequently far from being verified. The source of the discrepancy must be in fitting the cooling to a single exponential. The differences between experimental and theoretical values in Table 2 show that the cooling is actually an exponential law, but there were slight deviations in the first few values of the temperature. This means that a better fit could result from trying a double exponential law

$$T = T_i' \mathrm{e}^{-m't} + T_i'' \mathrm{e}^{-m''t} \tag{13}$$

where the actual H will be obtained through the appropriate combination of the H values obtained from m' and m'', which may obey a law of the type of eqn. (8).

Table 3 is the same as Table 2 but for a double exponential, where the fit turns out to be quite good, as can be seen from the last column. The m coefficients are now very sensitive to the temperature, with a higher value H' obtained from m', which governs the sharper decay of the temperature, and a second smaller value H'' from m'', which controls the smoother temperature decay. In separate plots of $\ln(T/T_1')$ for shorter times and $\ln(T/T_1'')$ for longer times, m' and m'' would be the slopes of the respective straight lines, and the slope m_t for the whole time will be their difference. This behaviour is the same for the H coefficients. Table 4 shows these values: the standard deviation is ± 0.03 for m_t values and ± 1.0 for the values of H.

The values are very similar and only the third is appreciably higher. Here

TABLE 4

Values for the surface conductances obtained from the results in Table 3

Case	$-m_t \times 10^{-3}$	$-H'$	$-H''$	$-H$
II	0.30	20.4	5.9	14.5
III	0.29	17.9	3.8	14.1
VIII	0.41	27.6	7.6	20.0
IX	0.32	21.8	6.5	15.3

again, case VIII shows a discrepancy with respect to the rest. Nevertheless, the mean value for H is 16.0, in agreement, within statistical error, with that obtained in the steady state. The mean value of m_t is $\overline{m}_t = 0.33$, which is higher than the mean value obtained for the single exponential fit (0.24). The experimental values are now very close to the verification of the theoretical relationship given by eqn. (12). The exact verification of eqn. (12) would mean that eqn. (2) could be substituted by eqn. (6) because the second derivative of the temperature with respect to x would be zero, i.e. the body would be spacewise isothermal. This is an approximation whose goodness will depend on the characteristics of the body and the experimental conditions.

The above discussion for iron could open new ideas for the study of another type of material quite different from a metallic bar, such as amorphous or plastic media [5,6].

CONCLUSIONS

From the results obtained for the temperature distribution in the iron bar, determining the behaviour of the surface conductance H, it is possible to draw the following conclusions.

As can be seen in Table 1, the steady state in the iron bar obeys a single exponential decay for the temperature, eqn. (4), confirming one of the conclusions of ref. (3).

The surface conductance H, calculated from the exponent m in the steady state, is found to be a constant, in agreement with the linear hypothesis of eqn. (1), but with a fluctuation of about 8%.

The cooling of the iron bar obeys a double exponential, eqn. (13), with first and second exponents governing the sharper and the smoother temperature gradient, respectively. It means that there are two surface conductances, H' and H'', where the first controls the cooling at short times and the second, which is about 70–80% smaller, controls the cooling at longer times.

The difference of the two surface conductances obtained during cooling was found to be equal to that calculated for the steady state, and eqn. (12) was verified approximately. Some uncertainty could appear because of the hypothesis that the heat flow is controlled principally by convection resistance, giving rise to eqn. (6).

Among the nine cases studied, only case VIII shows slight discrepancies in the steady state and cooling, and this could mean that there could be some doubt about an unknown error during the experimental procedure or in reading the data.

Finally, in order to calculate the Biot modulus used in the cooling, eqns. (7) and (9), in a simple and accurate way, it may be better to calculate the value H from the steady state, rather than from the cooling, and to use the resulting value in eqn. (10).

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